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FORWARD DISPLACEMENT ANALYSIS OF GENERAL SIX-IN-PARALLEL SPS (STEWART) PLATFORM MANIPULATORS USING SOMA COORDINATES

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Abstract—General six-in-parallel SPS platform manipulators are constructed of six telescoping legs, each connecting a stationary base platform to a moving platform via spherical joints. These are often termed "generalized Stewart platforms." Given the lengths of the six legs, the forward displacement problem is to find the location of the end platform relative to the base platform. It was first demonstrated numerically that the problem may in general have at most 40 nonsingular solutions and this bound has been verified using several different mathematical arguments. The problem is reformulated in this report using a classical representation of rigid-body displacements: Study's soma coordinates, or equivalently, dual quaternions. This provides a much simpler analytical proof of the upper bound of 40. Moreover, the simple form of the equations may be useful in further studies of the problem.

1. INTRODUCTION

Consider a platform manipulator as shown in Fig. 1, consisting of a moving "end plate" supported from a "base plate" by six extensible "legs". On the end plate there have been selected six points having vector coordinates $b_0, \ldots, b_5$ in the reference frame of the body. For each of these there is a corresponding point in the base plate having vector coordinates $a_0, \ldots, a_5$, resp., in the fixed reference frame. At any particular position p and rotation R of the end plate, there will be unique squared distances

$$L_i^2 = (p + Rb_i - a_i)^{\mathrm{T}}(p + Rb_i - a_i), \quad i = 0, \ldots, 5. \tag{1}$$

One may build a device wherein the distances L_i are directly actuated by a prismatic joint in line with two spherical joints connected at points a_i and b_i. When held at constant length, each such leg imposes one constraint on the motion of the end plate, and hence six legs together are generally sufficient to hold the end plate rigidly. Such a structure can be classified as a six-in-parallel SPS platform manipulator, also known as a generalized Stewart platform.

The inverse displacement problem of determining the joint displacements L_i necessary to hold the end plate in a given location (p, R) are straightforwardly evaluated from equation (1). However, the forward displacement problem of determining the end plate location given the leg lengths is more difficult. In planning and control of such a robot, it is convenient to have efficient solution algorithms for both problems.

It is known that the forward displacement problem has at most 40 nonsingular solutions. This was first demonstrated numerically, using polynomial continuation [1]. When carried out carefully, this approach provides a strong experimental demonstration, with solid mathematical underpinnings. Also, polynomial continuation can provide a reasonably efficient method for determining solutions in floating point arithmetic. For example, a 7-variable formulation that tracks 40 continuation paths has been timed at approximately 14 s CPU time on an IBM RS/6000 workstation in Fortran double precision, with proportionately less CPU time required to solve platforms of various special geometries having fewer solutions [2]. Other applications of continuation to this problem are reported in [3, 4].

Even with a trustworthy numerical demonstration in hand, a rigorous mathematical proof is

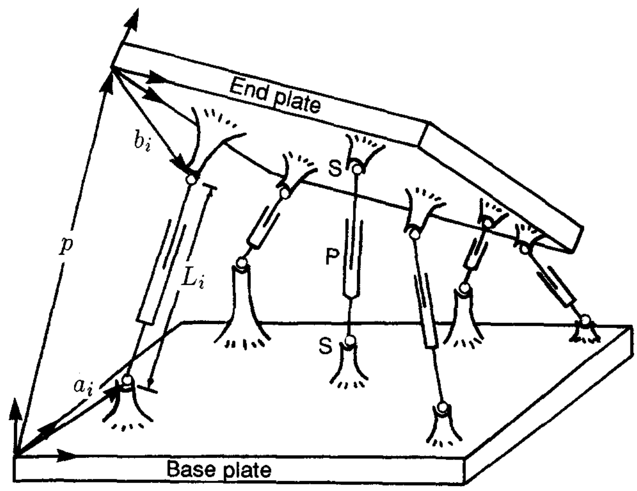

Fig. 1. A general six-in-parellel SPS platform manipulator.

desirable to eliminate all doubt and possibly to better understand the problem. It could be hoped that such understanding might lead to better numerical algorithms for solving the problem. The first proof has been attributed by Lazard [5] to Ronga and Vust (unpublished). Unfortunately, the proof was of an abstract mathematical nature (intersection theory, Chern classes, etc.), which is of little assistance to the roboticist in the search for better algorithms. An alternative proof due to Lazard [5] hinges on the computation of a Gröbner basis for rigid-body motions represented by vectors p, t and rotation matrix R, subject to the following polynomial relations:

$$R^{\mathrm{T}}R = I, \quad \det R = 1, \quad p = Rt, \quad t = R^{\mathrm{T}}p. \tag{2}$$

The resulting Gröbner basis has 41 elements, 5 of which are cubic with the rest being quadratics. It is claimed that computing a Hilbert function of this ideal shows that it is of degree 20, and therefore with the addition of the element $p^{\mathrm{T}}p$ we have degree 40. Equations (1) are linear on these elements, hence there are at most 40 nonsingular solutions. This proof suggests the use of Gröbner bases to solve particular examples. However it was found that when an example problem of the form of equations (1) was appended to the basis, the computation of the new Gröbner basis required on the order of 4 hours [5]. Lazard's Gröbner basis and Hilbert function calculations were done by computer. Mourrain [6] has put forward a similar demonstration worked out by hand, although it is not an easy derivation. Mourrain has also attacked the problem using quaternions to represent rotation [7]. He reports obtaining a Gröbner basis of degree 40 for specific examples in about 1/2 hour. Furthermore, he gives an argument using resultants which shows that, after properly accounting for solutions at infinity, the general case has 40 solutions.†

In addition to these results for the general case, there have been many publications concerning platforms with special geometries wherein the problem has been reduced to a polynomial of degree 40 or less in one variable [1, 8–22]. Such reductions generally lead to the fastest algorithms, although issues of numerical stability have not been closely examined.

The contribution of this paper is to give a simple proof of the upper bound of 40 nonsingular solutions. We formulate the problem in soma coordinates, which leads to a simpler presentation of the polynomial system. From this new formulation, the upper bound on the number of solutions can be concluded by examining the degrees of the equations. It is hoped that the simplicity of this new formulation will be useful in further understanding of the forward displacement problem.

† Subsequent to the submission of this paper, a reduction of the problem to a univariate polynomial of degree 40 has been found by M. L. Husty [23].

2. FORMULATION IN SOMA COORDINATES

Study's soma coordinates are, except for a factor of 1/2, identical to dual quaternions [24, pp. 150–152, pp. 521–524]. We recall the following facts concerning quaternions. First, we note that a quaternion is a 4-tuple, say $q = (q_0, q_1, q_2, q_3)$, which we write as a scalar q_0 and a vector $\hat{q} = (q_1, q_2, q_3)$, hence $q = (q_0, \hat{q})$. We speak of the *real part* of a quaternion, $\mathrm{Re}(q) = q_0$, and also the *conjugate* of a quaternion, $q' = (q_0, -\hat{q})$. A scalar by itself may be considered a quaternion whose vector part is zero, and similarly a vector standing alone is a quaternion whose real part is zero. Addition of quaternions is carried out element by element, in the natural way. Quaternion multiplication, which we denote with the symbol "*", can be expressed in terms of vector dot-products and cross-products in 3-space as

$$q * r = (q_0 r_0 - \hat{q} \cdot \hat{r}, \hat{q} \times \hat{r} + q_0 \hat{r} + \hat{q} r_0). \tag{3}$$

Other useful facts are that quaternion multiplication is associative, and conjugation of a product behaves like a matrix transpose, i.e., $(q * r)' = r' * q'$. An important fact for our purposes is that for any 3×3 rotation matrix R there is a corresponding unit quaternion e such that the rotation Rb of an arbitrary vector b is given by

$$e * e' = 1, \tag{4}$$

$$Rb = e * b * e'. \tag{5}$$

In this representation, e and $-e$ both give the same rotation R. The right-hand side of equation (5) can be written out in matrix form to give an explicit formula for R in terms of the components of the quaternion.

In addition to introducing the quaternion e to represent rotation, soma coordinates include a quaternion g to indirectly represent position as

$$p = g * e'. \tag{6}$$

Since the position p is a pure vector, we have the requirement

$$\mathrm{Re}\,(g * e') = 0. \tag{7}$$

Note that the pairs (g, e) and $(-g, -e)$ both give the same rotation and position. Whenever (g, e) satisfy the additional conditions equations (4, 7), they give a unique position and rotation as in equations (5, 6). Conversely, except for the choice of sign, a given rotation R determines a unique, non-zero e, from which we may find g as $g = p * e$. Thus, the soma coordinates, (g, e) subject to equations (4, 7), form a set of coordinates for representing rigid-body displacements.

With these preparatory facts in hand, we may restate the forward kinematics problem in soma coordinates. Using equations (5, 6) to rewrite Rb_i and p, and using the fact that multiplication of a quarternion by its own conjugate gives the squared length, one obtains

$$L_i^2 = (g * e' + e * b_i * e' - a_i) * (g * e' + e * b_i * e' - a_i)', \quad i = 0, \ldots, 5. \tag{8}$$

These six equations along with equations (4, 7) are 8 equations to be solved for (g, e).

3. REDUCTION TO QUADRATIC EQUATIONS

We begin by expanding equation (8) and use the fact that $q + q' = 2\,\mathrm{Re}(q)$ for any quaternion q to get for $i = 0, \ldots, 5$

$$\begin{aligned} L_i^2 = {} & g * e' * e * g' + e * b_i * e' * e * b_i' * e' \\ & + a_i * a_i' + 2\,\mathrm{Re}(g * e' * e * b_i' * e' - g * e' * a_i' - e * b_i * e' * a_i'). \end{aligned} \tag{9}$$

Since e is a unit quaternion, this can be rewritten as

$$g * g' + (b_i * b_i' + a_i * a_i' - L_i^2)(e * e') + 2\,\mathrm{Re}(g * b_i' * e' - g * e' * a_i' - e * b_i * e' * a_i') = 0. \tag{10}$$

(We could replace $e * e'$ with 1, but write the equations in this *homogeneous* form to assist in further reduction below.) Without loss of generality, we may choose a_0 and b_0 as the origin points for the base and end coordinate systems, i.e., let $a_0 = 0$ and $b_0 = 0$. Accordingly,

$$g * g' - L_0^2 e * e' = 0. \tag{11}$$

This equation can be subtracted from the other five to get

$$(b_i * b_i' + a_i * a_i' - L_i^2 + L_0^2)(e * e') + 2\,\mathrm{Re}(g * b_i' * e' - g * e' * a_i' - e * b_i * e' * a_i') = 0, \quad i = 1, \ldots, 5. \tag{12}$$

Equations (7, 11, 12) are 7 homogeneous quadric equations in $(g, e) \in \mathbf{P}^7$, where $\mathbf{P}^7$ is seven-dimensional projective space, i.e., lines through the origin in $\mathbf{C}^8$. The total degree is $2^7 = 128$. Alternatively, equations (4, 7, 11, 12) are 8 quadric equations in $(g, e) \in \mathbf{C}^8$ with total degree 256. Each solution in $\mathbf{P}^7$ cuts the unit sphere [equation (4)] in two diametrically opposed points, which are the solutions in $\mathbf{C}^8$. Of course, in either case, even though we may choose to solve the polynomial equations over complex numbers, only real solutions are physically meaningful.

In what follows, we abandon the quaternion notation and rewrite the equations considering g and e as 4×1 column vectors. One may check that the equations are of the form

$$e^{\mathrm{T}} e = 1 \tag{13}$$

$$g^{\mathrm{T}} e = 0 \tag{14}$$

$$g^{\mathrm{T}} g - L_0^2 e^{\mathrm{T}} e = 0 \tag{15}$$

$$e^{\mathrm{T}} A_i e + 2 g^{\mathrm{T}} B_i e = 0, \quad i = 1, \ldots, 5, \tag{16}$$

where A_i and B_i are 4×4 matrices depending on a_i, b_i, L_i and L_0. These are given explicitly in the Appendix. It is important to note that B_i is antisymmetric. This may be shown using the facts that $\mathrm{Re}(q) = \mathrm{Re}(q')$ and $\mathrm{Re}(q * r) = \mathrm{Re}(r * q)$ for any quaternions q and r. Consequently,

$$g^{\mathrm{T}} B_i e = \mathrm{Re}(g * b_i' * e' - g * e' * a_i') = \mathrm{Re}(e * b_i * g' - a_i * e * g') = \mathrm{Re}(e * b_i * g' - e * g' * a_i).$$

But a_i and b_i are pure vectors, so $a_i' = -a_i$ and $b_i' = -b_i$. Thus,

$$g^{\mathrm{T}} B_i e = -\mathrm{Re}(e * b_i' * g' - e * g' * a_i') = -e^{\mathrm{T}} B_i g = -g^{\mathrm{T}} (B_i)^{\mathrm{T}} e.$$

Since this holds for any g and e, we have $B_i = -(B_i)^{\mathrm{T}}$, which is the result we seek.

It is convenient to make g non-dimensional, which can be accomplished with the change of coordinates $g = L_0 \bar{g}$. We get

$$e^{\mathrm{T}} e = 1 \tag{17}$$

$$\bar{g}^{\mathrm{T}} e = 0 \tag{18}$$

$$\bar{g}^{\mathrm{T}} \bar{g} - e^{\mathrm{T}} e = 0 \tag{19}$$

$$e^{\mathrm{T}} \tilde{A}_i e + 2 \bar{g}^{\mathrm{T}} B_i e = 0, \quad i = 1, \ldots, 5, \tag{20}$$

where $\tilde{A}_i = A_i / L_0$. It can be shown using intersection theory and Chern classes that ignoring the first of these equations and working on $\mathbf{P}^7$, this system has at most 84 isolated solutions when the B_i are general, and at most 40 isolated solutions when the B_i are antisymmetric [25]. However, we wish to proceed in a more elementary way.

4. PROOF OF 40 SOLUTIONS

Theorem. For any $\tilde{A}_i$ and any antisymmetric B_i, $(i = 1, \ldots, 5)$, the polynomial system of equations (17–20) has at most 40 pairs of nonsingular solutions of the form $\pm(\bar{g}, e)$.

Proof: The fact that the solutions appear in pairs differing only in sign is readily seen by noting that every term appearing in the equations is of degree 0 or 2. We will establish the upper bound of 40 pairs by writing the system as a member of a parameterized family of systems and showing that no member of that family can have more than 80 nonsingular solutions. To this end, consider the modified system consisting of equations (17–19) with

$$e^{\mathrm{T}} \tilde{A}_i e + 2 \bar{g}^{\mathrm{T}} B_i e - \lambda^2 \bar{g}^{\mathrm{T}} \tilde{A}_i \bar{g} = 0, \quad i = 1, \ldots, 5. \tag{21}$$

The forward kinematic problem is the same as this new system when $\lambda^2 = 0$. Suppose, however, that we solve this for some other value $\lambda \neq 0$. Then, we may use the change of coordinates

$$x = e + \lambda\bar{g}, \quad \hat{x} = e - \lambda\bar{g}. \tag{22}$$

Due to the quadratic form of the equations, we may assume without loss of generality that $\tilde{A}_i$ is symmetric. Further, since B_i is antisymmetric, it can be seen that

$$x^{\mathrm{T}}(\tilde{A}_i + B_i/\lambda)\hat{x} = e^{\mathrm{T}}\tilde{A}_i e + 2\bar{g}^{\mathrm{T}}B_i e - \lambda^2\bar{g}^{\mathrm{T}}\tilde{A}_i\bar{g}, \tag{23}$$

and hence by equation (21)

$$x^{\mathrm{T}}(\tilde{A}_i + B_i/\lambda)\hat{x} = 0, \quad i = 1, \ldots, 5. \tag{24}$$

Additionally, using equations (17–19), we have

$$x^{\mathrm{T}}\hat{x} = e^{\mathrm{T}}e - \lambda^2\bar{g}^{\mathrm{T}}\bar{g} = 1 - \lambda^2, \tag{25}$$

$$x^{\mathrm{T}}x = e^{\mathrm{T}}e + \lambda^2\bar{g}^{\mathrm{T}}\bar{g} + 2e^{\mathrm{T}}\bar{g} = 1 + \lambda^2, \tag{26}$$

$$\hat{x}^{\mathrm{T}}\hat{x} = e^{\mathrm{T}}e + \lambda^2\bar{g}^{\mathrm{T}}\bar{g} - 2e^{\mathrm{T}}\bar{g} = 1 + \lambda^2. \tag{27}$$

The 2-homogeneous Bezout number of the transformed system, equations (24–27), can be computed as follows [26]. Equations (24, 25) are bi-linear in x and $\hat{x}$, equation (26) is quadratic in x, and equation (27) is quadratic in $\hat{x}$. Accordingly, the Bezout number is the coefficient of $\alpha^4\beta^4$ in the combinatorial product $(\alpha + \beta)^6(2\alpha)(2\beta)$, which is $\binom{6}{3}2^2 = 80$. This shows that equations (24–27) can have at most 80 nonsingular solutions. For $\lambda \neq 0$, variables $(x, \hat{x})$ are a nonsingular linear change of coordinates from variables $(\bar{g}, e)$, and thus the system of equations (17–19, 21) also has at most 80 nonsingular solutions.

By Theorem 1 of [27], the number of nonsingular solutions for generic λ is an upper bound on the number of nonsingular solutions for any particular λ, and so equations (17–19, 21) can have at most 80 such solutions at $\lambda = 0$. But this is the same system as the original system, equations (17–20). Since solutions to that system appear in pairs $\pm(\bar{g}, e)$ corresponding to the same physical displacement, there are at most 40 solutions to the forward displacement problem. Q.E.D.

Remark 1. Although we have proven only that 40 is an upper bound, it is in fact a sharp bound. Any demonstration of an example having 40 solutions is sufficient to establish this result, which has been done both by numerical continuation and in exact arithmetic via Gröbner bases (see Section 1). An example having 40 real solutions has yet to be found.

Remark 2. Rather than appealing to Theorem 1 of [27], the proof can be based on a continuity argument. By continuity and the implicit function theorem, each nonsingular solution at $\lambda = 0$ extends to a nonsingular solution on an open set around zero. But we have bounded the number of nonsingular solutions for $\lambda \neq 0$.

Remark 3. The proof suggests a homotopy for solving the problem by polynomial continuation. Choose generic complex λ and $(\tilde{A}_i + B_i/\lambda) = u_i v_i^{\mathrm{T}}$, where u_i and v_i are generic complex 4×1 vectors. Then, equation (24) is the product of two linear factors: $(u_i^{\mathrm{T}}x)^{\mathrm{T}}(v_i^{\mathrm{T}}\hat{x}) = 0$. For each $i = 1, \ldots, 5$, one of these factors must be zero. All 40 pairs of solutions to equations (24–27) can then be found via Gaussian elimination and the quadratic formula. By the definition of x, $\hat{x}$, these can be transformed to pairs of solutions to equations (17–19, 21). Tracking one of each pair of solution paths in a homotopy taking λ to zero and $\tilde{A}_i$, B_i to the target problem will form a valid 40-path continuation method for obtaining all non-singular solutions to the forward displacement problem. However, this does not offer any significant advantage over a more conventional parameter homotopy wherein one solves equations (17–20) *once* as a 256-path homotopy to a generic example and then thereafter tracks 40 paths in a homotopy through the $\tilde{A}_i$, B_i-parameter space.

Remark 4. The transformed coordinates $x = e + \lambda\bar{g}$ are strongly reminiscent of displacements formulated in terms of the "dual quaternion" $e + \epsilon g/2$ wherein ϵ is the "dual" number $(\epsilon \neq 0, \epsilon^2 = 0)$ [24, pp. 521–524].

5. CONCLUSIONS

The forward displacement problem can be expressed in soma coordinates as either 8 quadric equations on $\mathbf{C}^8$ or 7 homogeneous quadric equations on $\mathbf{P}^7$. In either case, it is shown that there are at most 40 nonsingular solutions using the principles of parameter continuation and 2-homogeneous Bezout numbers. The argument is much simpler than previous proofs. Moreover, the low degree and sparsity of the polynomials suggested that this may be a good starting point for future work on this problem.

The aptness of soma coordinates for this problem suggests that they may also be useful in solving the kinematics of other in-parallel linkages or mixed parallel/serial linkages.

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APPENDIX

Expressions for A_i and B_i

Let I_4 and I_3 be the 4×4 and 3×3 identity matrices, respectively. The matrices A_i and B_i derive from equation (14), which is in quaternion form. Using the formula for quaternion multiplication as in equation (3) to expand equation (14) and noting that a_i and b_i are both pure vectors, one obtains

$$A_i = (b_i^{\mathrm{T}} b_i + a_i^{\mathrm{T}} a_i - L_i^2 + L_0^2) I_4 - 2\begin{pmatrix} a_i^{\mathrm{T}} b_i & (b_i \times a_i)^{\mathrm{T}} \\ b_i \times a_i & b_i a_i^{\mathrm{T}} + a_i b_i^{\mathrm{T}} - a_i^{\mathrm{T}} b_i I_3 \end{pmatrix}.$$

To write B_i in matrix form it is convenient to introduce the following notation: for any 3×1 vector v, let $\Lambda(v)$ denote the antisymmetric 3×3 matrix corresponding to the cross-product, that is $\Lambda(v)x = v \times x$. Then one has

$$B_i = \begin{pmatrix} 0 & (a_i - b_i)^{\mathrm{T}} \\ b_i - a_i & -\Lambda(a_i + b_i) \end{pmatrix}.$$

ANALYSE DU MODÈLE GÉOMÉTRIQUE DIRECT DES ROBOTS PARALLÈLES GÉNÉRAUX À L'AIDE DES COORDONNÉES "SOMA"

Résumé—Les robots parallèles généraux sont constitués de six segments de longueur variable reliant une base fixe à un plateau mobile. Chaque segment est connecté à la fois à la base et au plateau mobile par des rotules. Ces robots sont parfois appelés "Plate-forme de Stewart généralisée". Le modèle géométrique direct consiste à déterminer la pose du plateau mobile étant donné les longueurs des six segments. Il a d'abord été démontré numériquement que ce problème ne pouvait avoir plus de 40 solutions et cette borne a par la suite été confirmée par des preuves utilisant divers arguments mathématiques. Dans cet article le problème est formulé à l'aide d'une représentation classique des déplacements d'un corps solide: les coordonnées "Soma" définies par Study qui sont équivalents aux quaternions duaux. Cette représentation permet d'établir une preuve analytique beaucoup plus simple de la borne du nombre de solutions. De plus la forme simplifiée qui est obtenue pour les équations peut s'avérer utile pour la suite de l'analyse de ce problème.